Q.1 The Schrödinger equation for a free electron of mass m and energy E written in terms of the wavefunction \( \psi \) is 
\[
\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m E}{\hbar^2} = 0.
\]
The dimensions of the coefficient of \( \psi \) in the second term must be -
(A) \([M^{-1} L^1]\)  
(B) \([L^2]\) 
(C) \([L^2]\)  
(D) \([M^{-1} L^1 T^{-1}]\)

Sol. [C]

By dimensional analysis the dimensions of each term in an equation must be the same. In the first term the second derivative with respect to distance x indicates the dimensions of the coefficient of \( \psi \) to be \([L^{-2}]\) and hence the answer.

Q.2 In an experiment to verify Poiseuille’s law
\[
\eta = \frac{\pi \Delta p a^4}{8V}
\]
where the symbols have their usual meanings, the maximum error that enters in calculating the coefficient of \( \eta \) is due to measurement of -
(A) pressure difference \( \Delta p \)  
(B) length of the tube \( l \)  
(C) volume rate of flow \( V \)  
(D) radius of the tube \( a \).

Sol. [D]

Since \( a \) is raised to the power 4, it has the largest contribution to the total error.

Q.3 A uniform rope of length L and mass M partly lies on a horizontal table and partly hangs from the edge of the table. If \( \mu \) is the coefficient of friction between the rope and the surface of the table (neglecting the friction at the edge), the maximum fraction of the length of the rope that overhangs from the edge of the table without sliding down is:

(A) \( \frac{1 - \mu}{\mu} \)  
(B) \( \frac{\mu}{\mu + 1} \)  
(C) \( 1 - \mu \)  
(D) \( \frac{1}{\mu + 1} \)

Sol. [B]

If \( l \) is the length of the hanging part of the rope, its weight is \( \left( \frac{M}{L} \right) l \times g \) and hence the weight of the rope on the table is \( \left( \frac{M}{L} \right) (L - l) g \) which is also equal to the normal reaction \( N \). Then, the force of friction is \( \mu N = \mu \left( \frac{M}{L} \right) (L - l) g \).

Equating this to the weight of the hanging part of the rope, we get the answer.

Q.4 A thin uniform rod XY of length 2l is hinged at one end X to the floor and stands vertical. When allowed to fall, the angular speed with which the rod strikes the floor is -

(A) \( \sqrt{\frac{3g}{4l}} \)  
(B) \( \sqrt{\frac{3g}{2l}} \)  
(C) \( \sqrt{\frac{3g}{l}} \)  
(D) \( \sqrt{\frac{g}{2l}} \)

Sol. [B]

The moment of inertia of the rod about a horizontal axis passing through point X is
\[
I_x = \frac{1}{12} m(2l)^2 = \frac{4}{3} m l^2
\]
where \( m \) is the mass of the rod. Now, since centre of gravity falls through a height \( l \), the change in gravitational potential energy \( (mgl) \) of the rod can be equated to the rotational energy
\[
\frac{1}{2} I_x \omega^2
\]
where \( \omega \) is the angular speed.

Q.5 A 60 cm metal rod \( M_1 \) is joined to another 100 cm metal rod \( M_2 \) to form an L shaped single piece. This piece is hung on a peg at the joint. The two rods are observed to be equally inclined to the vertical. If the two rods are equally thick, the ratio of density of \( M_1 \) to that of \( M_2 \) is

(A) 5/3  
(B) 3/5  
(C) 25/9  
(D) 9/25
Sol. [C]
The angle made by each rod with the vertical is 45°. Applying the law of moments, we get the ratio of weights and hence that of masses to be \(\frac{5}{3}\). Then, using mass = volume × density, we get the ratio of densities as 25 : 9.

Q.6
The string of a simple pendulum is replaced by a thin uniform rod of mass M and length \(l\). The mass of the bob is \(m\). If it is allowed to oscillate with small amplitude, the period of oscillation is

(A) \(2\pi \sqrt{\frac{2(M+3m)L}{3(M+2m)g}}\)
(B) \(2\pi \sqrt{\frac{3(M+2m)L}{2(M+3m)g}}\)
(C) \(2\pi \sqrt{\frac{(M+2m)L}{(M+3)m}g}\)
(D) \(2\pi \sqrt{\frac{2(M+m)L}{3(M+2m)g}}\)

Sol. [A]
The restoring torque can be written as

\[- (M + 2m)\left(\frac{L}{2}\right) \theta \] which reduces to

\[- M g \left(\frac{L}{2}\right) \sin \theta + m g L \sin \theta \] if \(\theta\) is small. This is equated to \(I \alpha\) where the moment of inertia

\[I = \frac{1}{3} M L^2 + mL^2 = \frac{L^2}{3} (M + 3m)\]. After substitution, we get \(\alpha = \left(\frac{3(M + 2m)g}{2(M + 3m)L}\right) \theta\).

Comparing with \(\alpha = -(\sigma)^2 \theta\), expression for angular frequency and hence period can be obtained.

Q.7
Two stars each of mass \(m\) and radius \(R\) approach each other to collide head-on. Initially the stars are at a distance \(r\) (\(>> R\)). Assuming their speeds to be negligible at this distance of separation, the speed with which the stars collide is

(A) \(\sqrt{\frac{Gm \left(\frac{1}{R} - \frac{1}{r}\right)}{r}}\)
(B) \(\sqrt{\frac{Gm \left(\frac{1}{2R} - \frac{1}{r}\right)}{r}}\)
(C) \(\sqrt{\frac{Gm \left(\frac{1}{R} + \frac{1}{r}\right)}{r}}\)
(D) \(\sqrt{Gm \left(\frac{1}{2R} + \frac{1}{r}\right)}\)

Sol. [B]
The change in potential energy can be equated to the gain of total kinetic energy:

\[- \frac{Gmm}{r} - \left(- \frac{Gmm}{2R}\right) = 2 \left(\frac{1}{2} m v^2\right)\]

Q.8
A nonviscous liquid of density \(\rho\) is filled in a tube with \(A\) as the area of cross section, as shown in the figure. If the liquid is slightly depressed in one of the arms, the liquid column oscillates with a frequency

\[\frac{\rho A \sin \theta_1 - \sin \theta_2}{2}\]

(A) \(\frac{1}{2\pi}\)
(B) \(\frac{\rho A (\sin \theta_1 - \sin \theta_2)}{2m}\)
(C) \(\frac{\rho A (\sin \theta_1 - \sin \theta_2)}{2m}\)
(D) \(\frac{\rho A \sin \frac{\theta_1 + \theta_2}{2}}{2m}\)

Sol. [C]
The force that is responsible for restoring the liquid levels in the two arms of the tube is \(- \Delta p A = -(h_1 + h_2) \rho g A\) where \(\Delta p\) is the pressure difference and \(A\) is the area of cross section of the tube, \(h_1\) and \(h_2\) being the rise and fall of liquid levels in the two arms in vertical direction respectively. Note that the change in length of the liquid thread along the tube will be the same, say \(x\). Using this the force can further be written as \(- (x \sin \theta_1 + x \sin \theta_2) \rho g A\).

Writing force as mass times acceleration, we get expression for period and then for frequency.

Q.9
A wax candle floats vertically in a liquid of density twice that of wax. The candle burns at the rate of 4 cm/hr. Then, with respect to the surface of the liquid the upper end of the candle will

(A) fall at the rate of 4 cm/hr
(B) fall at the rate of 2 cm/hr
(C) rise at the rate of 2 cm/hr
(D) remain at the same height
Let the decrease in the height of the candle outside the liquid be \( x \). Now, if \( d \) denotes the diameter and \( L \) the length of the candle respectively, after one hour applying the law of flotation gives

\[
\pi \left( \frac{d}{2} \right)^2 (L - 4) \rho_{\text{wax}} = \pi \left( \frac{d}{2} \right)^2 \left( \frac{L}{2} - x \right) \rho_{\text{liq}}.
\]

Note that \( \rho_{\text{wax}} = \frac{1}{2} \rho_{\text{liq}} \) makes only half of the candle stand outside the liquid. Solving the equation one gets the answer.

Q.10

A mass hangs at the end of a massless spring and oscillates up and down at its natural frequency \( f \). If the spring is cut at the midpoint and the mass reattached at the end, the frequency of oscillation is:

(A) \( \sqrt{2} f \)  (B) \( 2 f \)  (C) \( f/2 \)  (D) \( f/\sqrt{2} \)

Sol. [A]

If \( k \) is the spring constant for the original spring and \( k' \) that for the half-cut spring, we have \( k' = 2k \). Use the expression for frequency

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

to get the answer.

Q.11

The density of a solid at normal pressure is \( \rho \). When the solid is subjected to an excess pressure \( \rho' \). The density changed to \( \rho' \). If the bulk modulus of the solid is \( k \), then the ratio \( \frac{\rho'}{\rho} \) is

(A) \( 1 + \frac{\rho}{k} \)  (B) \( 1 + \frac{k}{\rho} \)  (C) \( \frac{\rho}{\rho + k} \)  (D) \( \frac{k}{\rho + k} \)

Sol. [A]

We use expression for density \( \rho = \frac{m}{V} \) so as to get \( \frac{d\rho}{\rho} = -\frac{1}{V} \frac{dV}{V} \). Substituting this in the expression for bulk modulus, we get \( k = -\frac{\rho}{\frac{dV}{V}} \). Since increase of pressure increases the density, \( d\rho = \rho' - \rho \). Using this we get \( \frac{\rho' - \rho}{\rho} = \frac{\rho}{k} \) and hence the answer

Q.12

A metal ball \( B_1 \) (density 3.2 g / cc) is dropped in water, while another metal ball \( B_2 \) (density 6.0 g / cc) is dropped in a liquid of density 1.6 g / cc. If both the balls have the same diameter and attain the same terminal velocity, the ratio of viscosity of water to that of the liquid is:

(A) 2.0  (B) 0.5  (C) 4.0  (D) indeterminate due to insufficient data

Sol. [B]

Use the expression for terminal velocity of a body falling in a viscous liquid \( v = \frac{2r^2(\rho - \sigma)g}{9\eta} \), where symbols have their usual meanings.

Q.13

A thin copper rod rotates about an axis passing through its end and perpendicular to its length with an angular speed \( \omega_0 \). The temperature of the copper rod is increased by 100°C. If the coefficient of linear expansion of copper is \( 2 \times 10^{-5} / \degree C \), the percentage change in the angular speed of the rod is

(A) –2%  (B) –4%  (C) –0.2 %  (D) –0.4 %

Sol. [D]

The moment of inertia of the rod is \( \frac{ml^2}{3} \). Let \( I_0, \omega_0 \) denote the initial moment of inertia and initial angular velocity respectively and \( I, \omega \) be the corresponding quantities after heating the rod. Applying conservation of angular momentum, we get

\[
\omega = \frac{I_0\omega_0}{I} = \left[ \frac{mI_0^2}{3} \frac{\omega_0}{\omega} \right]^{1/3}
\]

\[
= \frac{I_0\omega_0}{I} = \frac{mI_0^2}{3} \left( 1 + 2\alpha \theta \right)
\]

Thus the change is 0.4% and is negative.

Q.14

The internal energy of one gram of helium at 100 K and one atmospheric pressure is

(A) 100 J  (B) 1200 J  (C) 300 J  (D) 500 J

Sol. [C]

The helium molecule is monatomic and hence internal energy per molecule is \( \frac{3}{2} kT \). The energy per mole is therefore is \( \frac{3}{2} RT \). One gram of helium is one fourth mole and hence its energy is \( \frac{1}{4} \times \frac{3}{2} R \times 100 = 300 \) J, taking the value of \( R \) to be approximately 8 J/mole-K.
Q.15 Volume of a monatomic gas varies with its temperature as shown. The ratio of work done by the gas to the heat absorbed by it when it undergoes a process from A to B is

(A) 2/3  (B) 2/5  (C) 2/7  (D) 1/3

Sol. [B]
Since the curve passes through the origin,\( V \propto T \), the pressure being constant. Then, heat absorbed at constant pressure is \( dQ = n C_p \, dT \) whereas the change in internal energy is \( dU = n C_v \, dT \) where the symbols have their usual meanings. The work done \( dW = dQ – dU = n (C_p – C_v) \, dT \). This gives the ratio of work done to the heat absorbed as \( \frac{2}{7} \) and hence the result.

Q.16 A small glass bead of mass \( m \) initially at rest starts from a point at height \( h \) above the horizontal and rolls down the inclined plane \( AB \) as shown. Then it rises along the inclined plane \( BC \). Assuming no loss of energy, the time period of oscillation of the glass bead is

(A) \( \sqrt{\frac{8h}{g}} (\sin \theta_1 + \sin \theta_2) \)
(B) \( \frac{14h}{5g} \left( \frac{1}{\sin \theta_1} + \frac{1}{\sin \theta_2} \right) \)
(C) \( \frac{8h}{g} \left( \frac{1}{\sin \theta_1} + \frac{1}{\sin \theta_2} \right) \)
(D) \( \frac{8h}{5g} \left( \frac{1}{\sin \theta_1} + \frac{1}{\sin \theta_2} \right) \)

Sol. [A]
Equating the angular momentum \( (mv_0h) \) about the center of mass to l one gets the answer.

Q.17 Water at 20°C (coefficient of viscosity = 0.01 poise) flowing in a tube of diameter 1 cm with an average velocity of 10 cm/s has the Reynold number

(A) 500  (B) 1000  (C) 2000  (D) indeterminate due to insufficient data

Sol. [B]
Reynold number is given by \( N = \frac{\rho vD}{\eta} \) where \( \rho \) is the density, \( v \) is the velocity and \( D \) is the diameter of the tube. Use this to get the answer.

Q.18 A billiard ball is hit by a cue at a point distance \( h \) above the centre. It acquires a linear velocity \( v_0 \). Let \( m \) be the mass and \( r \) be the radius of the ball. The angular velocity acquired by the ball is

(A) \( \frac{5v_0h}{2r^2} \)  (B) \( \frac{2v_0h}{3r^2} \)  (C) \( \frac{2v_0h}{5r^2} \)  (D) \( \frac{2v_0h}{r^2} \)

Sol. [A]
Equating the angular momentum \( (mv_0h) \) about the center of mass to l one gets the answer.

Q.19 Two pipes are each 50 cm in length. One of them is closed at one end while the other is open at both ends. The speed of sound in air is 340 m/s. The frequency at which both the pipes can resonate is

(A) 680 Hz  (B) 510 Hz  (C) 85 Hz  (D) None of these

Sol. [B]
The speed with which a sphere rolling down an inclined plane reaches the bottom is

\[ v = \sqrt{\frac{2gh}{1 + \frac{1}{mr^2}}} = \left( \frac{10}{7}gh \right)^{1/2} \]

where symbols have their usual meanings. The acceleration is \( \frac{5g \sin \theta}{7} \). With this, the time taken by the glass bead to roll down turns out to be \( \sqrt{\frac{14h}{5g}} \left( \frac{1}{\sin \theta_1} \right) \). Similarly the time to go up the other inclined plane is \( \sqrt{\frac{14h}{5g}} \left( \frac{1}{\sin \theta_2} \right) \). Twice the sum of these two times is the required time of oscillation.
A pipe open only at one end and another one of equal length but open at both the ends have their fundamental frequencies n and 2n respectively. Again only odd harmonics are possible for a pipe open at one end whereas all harmonics are possible for a pipe open at both ends. Thus, it is impossible to have a common frequency at which they can resonate.

Q.20 The work involved in breaking a bigger size spherical oil drop into n smaller size identical droplets is proportional to

(A) \( \frac{2}{3} n^2 - 1 \)    (B) \( \frac{2}{3} n \)

(C) \( \frac{1}{n} - 1 \)    (D) \( \frac{1}{n} \)

Sol. [D]

Note that the work involved in the process of breaking a bigger drop into smaller drops is the change in surface area times the surface tension. If r is the radius of smaller drop and R that of the bigger one, then \( \frac{4}{3} \pi R^3 = n \left( \frac{4}{3} \pi r^3 \right) \) where n is the number of smaller drops. This gives \( r = \frac{R}{\sqrt[n]{n}} \). If T is the surface tension, then the work done will be \( W = (n4\pi r^2 - 4\pi R^2)T \). Substituting for r gives the expected proportionality.

Q.21 A train moving towards a hill at a speed of 72 km/hr sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km/hr. If the speed of sound in air is 340 m/s, the frequency heard by a man on the hill is

(A) 532.5 Hz    (B) 565.0 Hz

(C) 516.5 Hz    (D) None of the above

Sol. [A]

If \( n' \) represents the apparent frequency and n the actual one, then use the relation

\( n' = n \cdot \frac{(v \pm w)}{(v \pm w) - v_s} \)

where \( v \) is the velocity of sound, \( w \) is the velocity of wind and \( v_s \) that of the source. Note that in this case the observer at rest.

Q.22 A convex lens forms a real image with magnification \( m_1 \) on a screen. Now, the screen is moved by a distance \( x \) and the object is also moved so as to obtain a real image with magnification \( m_2 \) on the screen. The, the focal length of the lens is

(A) \( \frac{m_1}{m_2} \) \( x \)    (B) \( \frac{m_2}{m_1} \) \( x \)

(C) \( x (m_1 - m_2) \)    (D) \( \frac{x}{(m_2 - m_1)} \)

Sol. [D]

Use the usual lens formula. In the first case if \( v \) is the image distance, \( (v + x) \) is the image distance after the movement. The magnifications \( m_1 \) and \( m_2 \) in the two cases turn out to be \( \left( \frac{v - 1}{f} \right) \) and \( \left( \frac{v + x}{f} - 1 \right) \) respectively. This can be simplified to get the expression for f.

Q.23 Two metal wires of identical dimensions are connected in series. If \( \sigma_1 \) and \( \sigma_2 \) are the conductivities of the metals respectively, the effective conductivity of the combination is

(A) \( \sigma_1 + \sigma_2 \)    (B) \( \frac{\sigma_1 + \sigma_2}{2} \)

(C) \( \sqrt{\sigma_1 \sigma_2} \)    (D) \( \frac{2\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} \)

Sol. [D]

Use the expression for resistance in terms of conductivity \( R = \frac{I}{\sigma A} \) and note that the total resistance is \( (R_1 + R_2) \) and total length of the wires is \( 2l \).

Q.24 An alternating supply of 220 volt is applied across a circuit with resistance 22 ohm and impedance of 44 ohm. The power dissipated in the circuit is

(A) 1100 watt.    (B) 550 watt.

(C) 2200 watt.    (D) (2200/3) watt

Sol. [B]

The current in the circuit is \( \frac{V}{Z} = 5 \) A and hence the power consumed is \( (\frac{220}{3})^2 \) equal to 550 watt.
Q.25 If $T$ denotes the temperature of the gas, the volume thermal coefficient of expansion of an ideal gas at constant pressure is

(A) $T$  (B) $\frac{1}{T}$  (C) $T^2$  (D) $\frac{1}{T^2}$

Sol. [B]

The volume coefficient of expansion is given by $\frac{dV}{VdT}$. For an ideal gas at constant pressure, we write $pdV = nRdT$ and substitute for $p$ from the usual relation $pV = nRT$ to get the required expression for the coefficient.

Q.26 A coil having $N$ turns is wound in the form of a spiral with inner radius $a$ and outer radius $b$ respectively. When a current $I$ passes through the coil, the magnetic field at the centre is:

(A) $\frac{2\mu_0 NI}{a+b}$  (B) $\frac{\mu_0 NI}{\sqrt{ab}}$

(C) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$  (D) $\frac{\mu_0 NI}{2(b-a)}ln\left(\frac{a}{b}\right)$

Sol. [D]

Consider $dN$ to be the number of turns in between radii $r$ and $(r+dr)$ so that we get $dN = \frac{N}{b-a}dr$. The magnetic induction $dB$ due to these many turns at the centre is $\frac{\mu_0 I(dN)}{2r}$. After substituting for $dN$ and integrating between $a$ and $b$, we get the result.

Q.27 A circuit is arranged as shown. Then, the current from A to B is:

(A) $+500$ mA  (B) $+250$ mA  

(C) $-250$ mA  (D) $-500$ mA

Sol. [B]

Use superposition theorem. We get the potential difference between A and B to be $+3.75$ volt when source of $5$ volt is shorted, whereas $-1.25$ volt when source of $10$ volt is shorted. Therefore, when both the sources are working the net potential difference is $+2.5$ volt so that current is $250$ mA from A to B.

Q.28 Two identical thin rings, each of radius $a$ are placed coaxially at a distance $a$ apart. Let charges $Q_1$ and $Q_2$ be placed uniformly on the two rings. The work done in moving a charge $q$ from the centre of one ring to that of the other is

(A) zero  (B) $\frac{q\sqrt{2}}{4\pi\varepsilon_0 a}$ $(Q_1 - Q_2)$

(C) $\frac{q(\sqrt{2} - 1)}{4\pi\varepsilon_0 a\sqrt{2}} (Q_1 - Q_2)$  (D) $\frac{q(\sqrt{2} - 1)}{4\pi\varepsilon_0 a} (Q_1 - Q_2)$

Sol. [C]

The electrostatic potential at the centre of the first ring with charge $Q_1$ is due to charge $Q_1$ itself as well as due to charge $Q_2$ on the other ring. This turns out to be $\frac{1}{4\pi\varepsilon_0} \frac{Q_1}{a} + \frac{1}{4\pi\varepsilon_0} \sqrt{2}a$. Similarly, the electrostatic potential at the centre of the other ring is $\frac{1}{4\pi\varepsilon_0} \frac{Q_1}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{a}$. The difference between these potentials times the charge $q$ is the required word done.

Q.29 An equilateral triangular loop of wire of side $l$ carries a current $i$. The magnetic field produced at the circumcentre of the loop is

(A) $\frac{\mu_0 3\sqrt{3}i}{4\pi l}$  (B) $\frac{\mu_0 9i}{4\pi l}$

(C) $\frac{\mu_0 18i}{4\pi l}$  (D) $\frac{\mu_0 6i}{4\pi l}$

Sol. [C]

If $l$ is the side of the triangle, the distance of the circumcentre from each of the side of the triangle carrying a current $i$ is $\frac{\mu_0 i}{4\pi} \sin 60^\circ + \sin 60^\circ = \frac{\mu_0 6i}{4\pi l}$. Since the direction of magnetic field in each case is the same, three times this would be the total magnetic induction.
Q.30 Consider a double slit interference experiment. Let \( E_0 \) be the amplitude of the electric field of the waves starting from the slits. If \( \phi \) is the phase difference between the two waves reaching the screen, the amplitude of resultant electric field at a point on the screen is

(A) \( E_0 \cos \phi \)  
(B) \( E_0 \cos (\phi/2) \)  
(C) \( 2E_0 \cos (\phi/2) \)  
(D) \( 2E_0 \cos \phi \)

Sol. [C]

Consider the magnitudes of the electric fields reaching the screen to be \( E_0 \sin (\phi t) \) and \( E_0 \sin (\phi t + \phi) \). Then, the resultant electric field at the screen would be sum of the two, that is,

\[
2E_0 \cos \left( \frac{\phi}{2} \right) \sin \left( \phi t + \frac{\phi}{2} \right).
\]

Note that the amplitude of the resultant electric field is the coefficient of the sine function.

Q.31 In a double slit experiment, the coherent sources are spaced 2d apart and the screen is placed a distance D from the slits. If \( n \)th bright fringe is formed on the screen exactly opposite to a slit, the value of \( n \) must be

(A) \( \frac{d^2}{2\lambda D} \)  
(B) \( \frac{2d^2}{\lambda D} \)  
(C) \( \frac{d^2}{\lambda D} \)  
(D) \( \frac{d^2}{4\lambda D} \)

Sol. [B]

The fringe width in this case is \( \frac{\lambda D}{2d} \) and the \( n \)th bright fringe is formed at a distance \( d \) away from the centre. Therefore, \( d = n \frac{\lambda D}{2d} \) giving the value of \( n \).

Q.32 When two sound sources of the same amplitude but of slightly different frequencies \( n_1 \) and \( n_2 \) are sounded simultaneously, the sound one hears has a frequency equal to

(A) \( |n_1 - n_2| \)  
(B) \( \frac{n_1 + n_2}{2} \)  
(C) \( \sqrt{n_1n_2} \)  
(D) \( [n_1 + n_2] \)

Sol. [B]

The resulting sound wave has a frequency equal to half the sum of the individual frequencies. Note that the resulting intensity varies at the beat frequency equal to difference of the individual frequencies.

Q.33 Four identical mirrors are made to stand vertically to form a square arrangements as shown in a top view. A ray starts from the midpoint M of mirror AD and after two reflections reaches corner D. Then angle \( \theta \) must be

(A) \( \tan^{-1}(0.75) \)  
(B) \( \cot^{-1}(0.75) \)  
(C) \( \sin^{-1}(0.75) \)  
(D) \( \cos^{-1}(0.75) \)

Sol. [B]

Note that the ray starting from point M at an angle \( \theta \) reaches the corner D at the right along a parallel path. Refer to the figure. Let \( a \) be the length of the side, so that tan \( \theta = \frac{x}{a/2} = \frac{a}{a-y} \). Solving these equations one gets \( x = \frac{2a}{3} \) and hence \( \cot \theta = \frac{3}{4} \).

Q.34 The reflecting surfaces of two mirrors M₁ and M₂ are at an angle \( \theta \) (angle \( \theta \) between 0° and 90°) as shown in the figure. A ray of light is incident on M₁. The emerging ray intersects the incident ray at an angle \( \phi \). Then,

(A) \( \phi = \theta \)  
(B) \( \phi = 180^\circ - \theta \)  
(C) \( \phi = 90^\circ - \theta \)  
(D) \( \phi = 180^\circ - 2\theta \)

Sol. [D]

If \( x \) is the angle of incidence when the ray strikes the mirror M₁ and \( y \) be that for mirror M₂, then, using simple properties of triangle one gets \( \phi = 180^\circ - 2(x + y) \) and (Students are expected to draw the ray diagram and check.)
Q.35 An unpolarized light beam is incident on a surface at an angle of incidence equal to Brewster's angle. Then,
(A) the reflected and the refracted beams are both partially polarized
(B) the reflected beam is partially polarized and the refracted beam is completely polarized and are at right angles to each other
(C) the reflected beam is completely polarized and the refracted beam is partially polarized and are at right angles to each other
(D) both the reflected and the refracted beams are completely polarized and are at right angles to each other

Sol. [C]
Refer to the article on polarization by reflection when the ray is incident at Brewster's angle from any standard book.

Q.36 Switch S is closed at t = 0. After sufficiently long time an iron rod is inserted into the inductor L. Then, the light bulb

(A) glows more brightly
(B) gets dimmer
(C) glows with the same brightness
(D) gets momentarily dimmer and then glows more brightly

Sol. [B]
As the rod is inserted, inductance increases and hence the voltage across inductor increases. This caused a drop in the voltage across the bulb and hence it gets dimmer.

Q.37 In the circuit shown below, the current that flows from a to b when the switch S is closed is:

(A) – 1.5 A  (B) + 1.5 A
(C) + 1.0 A  (D) – 1.0 A

Sol. [B]
At every instant the ratio of the magnitude of the electric field to that of the magnetic field in an electromagnetic wave equals the speed of light.

Q.38 Two radioactive materials A and B have decay constants $5\lambda$ and $\lambda$ respectively. Initially both A and B have the same number of nuclei. The ratio of the number of nuclei of A to that of B will be $\frac{1}{e}$ after a time

(A) $\frac{1}{5\lambda}$  (B) $\frac{1}{4\lambda}$

(C) $\frac{5}{4\lambda}$  (D) $\frac{4}{5\lambda}$

Sol. [B]
Using the law of radioactive decay, one can write $\frac{N_A(t)}{N_A(0)} = \frac{N_B(t)}{N_B(0)} \exp(-\lambda t) = \exp(-5\lambda t)$. Solving this one gets the result.

Q.39 The radius of the hydrogen atom in its ground state is $a_0$. The radius of a 'muonic hydrogen' atom in which the electron is replaced by an identically charged muon with mass 207 times that of an electron is $a_{\mu}$ equal to

(A) $207a_0$  (B) $\frac{a_0}{207}$

(C) $\frac{a_0}{\sqrt{207}}$  (D) $a_0\sqrt{207}$

Sol. [B]
Use the expression for the first Bohr radius for hydrogen atom. This expression indicates that the radius is inversely proportional to the mass and hence the result.

Q.40 The instantaneous magnitudes of the electric field (E) and the magnetic field (B) vectors in an electromagnetic wave propagating in vacuum are related as

(A) $E = \frac{B}{c}$  (B) $E = cB$

(C) $E = \frac{B}{c^2}$  (D) $E = c^2B$

Sol. [B]
At every instant the ratio of the magnitude of the electric field to that of the magnetic field in an electromagnetic wave equals the speed of light.
Q.41 A monkey holds a light rope that passes over a smooth pulley. A bunch of bananas of equal mass as that of the monkey is attached to the other end of the rope. The monkey starts climbing the rope to get to the bananas. Then,
(A) the bananas also move up
(B) the bananas move downwards
(C) the distance between the monkey and the bananas decreases
(D) the distance between the monkey and the bananas remains constant.

Sol. [A, D]
Note that the masses of the monkey and the bunch of bananas are equal and the pulley is smooth.

Q.42 Consider the curve representing the Maxwell-Boltzmann speed distribution of gas molecules at some temperature. Let \( v_{\text{rms}} \), \( v_{\text{avg}} \) and \( v_{\text{mp}} \) be the rms, the average and the most probable speeds respectively. Then,
(A) the curve has a maximum at \( v_{\text{mp}} \)
(B) the area under the curve gives the total number of molecules of the gaseous system.
(C) \( v_{\text{rms}} > v_{\text{avg}} > v_{\text{mp}} \)
(D) \( v_{\text{avg}} < v_{\text{mp}} < v_{\text{rms}} \)

Sol. [A, B, C]
Note that \( v_{\text{rms}} = 1.73 \sqrt{\frac{kT}{m}} \), \( v_{\text{avg}} = 1.60 \sqrt{\frac{kT}{m}} \), \( v_{\text{mp}} = 1.41 \sqrt{\frac{kT}{m}} \) where the symbols have their usual meanings. The most probable speed \( v_{\text{mp}} \) is the speed at which the curve reaches the peak. The area under the curve is obviously the total number of molecules.

Q.43 A particle of mass \( m \) moves along a straight line under the action of a force \( f \) varying with time as \( f = f_0 \left[ 1 - \left( \frac{t - T}{T} \right)^2 \right] \) where \( f_0 \) and \( T \) are positive constants. Then
(A) the speed of the particle after a time \( 2T \) is \( \frac{4f_0T}{3m} \)
(B) after time interval of \( 3T \), the particle starts moving backwards
(C) between time instants 0 and \( 2T \), the acceleration first increases and then decreases.
(D) the particle stops at \( t = 3T \)

Sol. [A, B, C, D]
Use the given expression for force to get an expression for acceleration. Integrate this to get an expression for velocity. Unless otherwise stated about the initial conditions, the velocity turns out to be \( v = f_0 \left[ \frac{1}{3mT^2} \left( t^2 - T^2 \right) - t^3 \right] \). Use this to get the required results.

Q.44 A sound wave of angular frequency \( \omega \) travels with a speed \( v \) in a medium of density \( \rho \) and bulk modulus \( B \). Let \( k \) be the propagation constant. If \( p \) and \( A \) are the pressure amplitude and displacement amplitude respectively, then the intensity of sound wave is
(A) \( \frac{1}{2} \pi BkA^2 \)
(B) \( \frac{vp^2}{2B} \)
(C) \( \frac{p^2}{2\rho v} \)
(D) \( \frac{B^2p^2}{2\rho} \)

Sol. [A, B, C, D]
Intensity, by definition, is the energy flowing per unit area per unit time. The displacement amplitude is given by \( A = \frac{p}{Bk} \) where \( k = \frac{\omega}{v} \) is the propagation constant. The speed \( v = \sqrt{\frac{B}{\rho}} \). Use these relations to get the required expressions.

Q.45 Two conducting plates A and B are placed parallel to each other at a small distance between them. Plate A is given a charge \( q_1 \) and plate B is given a charge \( q_2 \). Then,
(A) the outer surfaces of A and B (not facing each other) get no charge
(B) the inner surfaces of A and B (facing each other) get all the charge
(C) the inner surfaces of A and B (facing each other) get equal and opposite charge of magnitude \( \frac{|q_1 - q_2|}{2} \)
(D) the outer surfaces of A and B (not facing each other) get charge of the same polarity and of magnitude \( \frac{|q_1 + q_2|}{2} \)
Let a charge $q'$ be present on the inner surface of plate A so that on its outer surface the charge is $(q_1 - q')$. Obviously a charge $-q'$ will get induced on the inner surface of the plate B and a charge $(q_2 + q')$ will move to its outer surface. With these charges, write the net electric field at a point inside the plate and equate it to zero. This relation can be simplified to get the value of $q'$ and hence the conclusions.

Q.46 A man with normal vision uses a magnifying lens focal length 10 cm. Then,
(A) magnification of any value is possible
(B) maximum magnification possible is 3.5
(C) minimum magnification possible is 2.5
(D) magnification depends upon the distance of the lens from the eye

Sol. [B, C, D]

In case of a microscope the magnification is
\[ \left(1 + \frac{D}{f}\right) \]
when the image is formed at the distance of distinct vision D. However if the image is formed at infinity, the magnification is simply\[ \frac{D}{f} \].

Q.47 An electromagnetic wave is traveling through a medium of refractive index $n_1$ and is incident at the boundary of a medium of refractive index $n_2$. If the wave reflects at the boundary,
(A) the wave undergoes a phase change of 180°, if $n_1 < n_2$
(B) the wave undergoes a phase change of 180°, if $n_1 > n_2$
(C) the wave undergoes no phase change, if $n_1 < n_2$
(D) the wave undergoes no phase change, if $n_1 > n_2$

Sol. [A, D]

Note that an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which it is traveling. However, there is no phase reversal if the case is opposite.

Q.48 In cyclotron (particle accelerator) an ion is made to travel successively along semicircles of increasing radii under the action of a magnetic field. The angular velocity of the ion is independent of
(A) speed of the ion (B) radius of the circle (C) mass of the ion (D) charge of the ion

Sol. [A, B]

Use the expression $qvB = mr\omega^2$, where $v = r\omega$. The symbols carry their usual meanings. This indicated that angular velocity is independent of the radius of the circular path and the speed of the ion.

Q.49 Physical quantities A and B have the same dimensions. Then,
(A) $A \pm B$ must be a meaningful physical quantity.
(B) $A \pm B$ may not be a meaningful physical quantity
(C) $A/B$ must be a dimensionless quantity
(D) both must be either scalar or vector quantities.

Sol. [B, C]

The ratio of two quantities having the same dimensions must necessarily be a dimensionless quantity. However, two quantities having the same dimensions may not add to necessarily give a meaningful quantity; for example, work and torque have the same dimensions but their addition is meaningless.

Q.50 A step voltage $V_0$ is applied to a series combination of R and C as shown. Then,

(A) after sufficiently long time $V_R = 0$
(B) as time passes $V_R$ decrease as $(1/t)$
(C) after 1 ms, $V_C = 6.3$ volt (approximately)
(D) initially current through R is 10 mA
Sol. \( [A, C, D] \)

After sufficiently long time since the charging current drops to zero, the drop across the resistance is zero. The time constant is \( 1 \text{k} \Omega \times 1 \mu \text{F} = 1 \text{ms} \). Hence according to the definition of time constant, the voltage across the capacitor would be about 63\% of the maximum, that is, 6.3 volt after 1 ms. Initial current is obviously \( (10 \text{volt/1K} \Omega) = 10 \text{mA} \).

**PART B**

**Marks : 60**

* All questions are compulsory.

* All questions carry equal marks

**Q.51** A cube of side 10 cm is rigidly joined to a thin rod of length 40 cm. The rod is pivoted at the other end so that the rod along with the cube is able to rotate freely about the pivot in a vertical plane. A bullet of mass 50 g, moving horizontally hits a point of the cube 5 cm from the lower end and gets embedded into it. Determine the speed of the bullet so that the system just rises to a horizontal position. (mass of the rod = 100 g, mass of the cube = 750 g)

**Sol.**

Note that the collision between the bullet and the cube is inelastic. Hence the kinetic energy is not conserved but the momentum is conserved. Equating the angular momentum of the bullet to that of the rod together with the cube about the pivot gives a relation angular frequency \( \frac{v}{7.49} \), where \( v \) is the speed of the bullet. Note that, in general, the moment of inertia of the rod of length \( l \) and mass \( m \) about the pivot is \( \frac{1}{3} ml^2 \) and that of the cube of mass \( m \) about an axis through its centre of mass is \( \frac{1}{6} ma^2 \) where \( a \) is the side of the cube. The total moment of inertia of the rod, the bullet and the pivot comes out to be \( 0.168 \text{kg-m}^2 \). After the collision the system rises through a height \((0.40 + 0.05) \text{ m} \). The gain in gravitational potential energy can then be equated to the loss of rotational kinetic energy. This gives \( \sigma = 6.646 \text{ s}^{-1} \). From this the speed of the bullet can be calculated to be 49.78 m/s.

**Q.52** Consider a uniform square plate of side \( 2l \) made of wood. A semicircular portion is cut and attached to the right as shown. Determine the centre of mass of the redesigned plate.

**Sol.**

For the semicircular plate of radius \( l \), the center of mass lies at a distance of \( \frac{4l}{3\pi} \) from the centre.

Taking \( \sigma \) to be the mass per unit area, the position of centre of mass of the remaining piece of the square would be at a distance of \( \frac{l}{3} (3\pi - 4) \) from the centre of the original square plate. Now, taking the centre of the original square to the origin, the centre of mass of the new structure can be determined. This turns out to be at a distance of \( \frac{l}{3} \) to the right of the origin.

**Q.53** Consider two long parallel and oppositely charged thick wires of radius \( d \) with their central axes separated by a distance \( D \) apart. Obtain an expression for the capacitance per unit length of this pair of wires.

**Sol.**

Note that when the two wires form a capacitor, the charges reside only on the inner side; positive on one and negative on the other. Let us consider a point \( P \) distance \( r \) from the axis of one wire. Using Gauss flux theorem, the electric field \( E_1 \) at \( P \) due to the positive charge (of surface charge density \( \sigma \)) on this wire of unit length is \( E_1 = \frac{\sigma(\pi d)}{\varepsilon_0} \Rightarrow E_1 = \frac{\sigma d}{\varepsilon_0 (2r)} \). The point \( P \) is at a distance \((D - r)\) from the axis of the other wire carrying negative charge. Again using Gauss theorem, the electric field \( E_2 \) can be written as \( E_2 = \frac{\sigma d}{\varepsilon_0 2(D - r)} \). Obviously the two fields are in the same direction so that net electric field is \( E = \frac{\sigma d [1 + \ln \left( \frac{D - d}{d} \right)]}{2\varepsilon_0 r} \). Integrate this between the limits \( d \) and \((D - d)\) to get the potential difference \( \frac{\sigma}{\varepsilon_0} d \ln \left( \frac{D - d}{d} \right) \). Then, the capacitance per unit length turns out to be \( \frac{\pi \varepsilon_0}{\ln \left( \frac{D - d}{d} \right)} \).

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**Q.54** Fermat's principle states that 'when light ray travels between two points, the path is the one that required the least time'. Use this principle to derive law of reflection regarding angle of incidence and angle of reflection. You may refer to the following figure.

![Diagram of light ray](image)

**Sol.** Consider the total distance d to be made up of x to the left of the point of incidence and (d – x) to its right. If n is the refractive index of the medium and c the speed of light in vacuum, then the speed in the medium under consideration is \( \frac{c}{n} \). The total time of travel can be written as

\[
t = \frac{x}{\sqrt{x^2 + h^2}} + \frac{n}{c} \sqrt{(d - x)^2 + h^2}.
\]

According to Fermat principle, for the least time, calculate \( \frac{dt}{dx} \) and equate it to zero. Using simple geometry, we write

\[
\frac{x}{\sqrt{x^2 + h^2}} = \sin \theta_1 \quad \text{and} \quad \frac{(d - x)}{\sqrt{(d - x)^2 + h^2}} = \sin \theta_2,
\]

we get the law of reflection that the angle of incidence is equal to the angle of reflection.

**Q.55** Two sample X and Y of a gas have equal volumes and pressure. The gas in X is allowed to expand isothermally to 1.5 times its initial volume, while that in Y is allowed to expand adiabatically to an equal volume. If the work done in the first expansion is 1.5 times that in the second, show that the ratio of specific heats \( \gamma \) satisfies a relation

\[
(\gamma - 1) \ln \left( \frac{3}{2} \right) = \frac{3}{2} \left[ 1 - \left( \frac{2}{3} \right)^{\gamma - 1} \right]
\]

**Sol.** Let \( n_1 \) and \( n_2 \) be the number of moles of samples X and Y of the gas, the initial temperatures of \( T_1 \) and \( T_2 \) respectively and \( p_0 \) and \( V_0 \) be their initial pressure and volume. Then, \( p_0V_0 = n_1RT_1 = n_2RT_2 \). Now, the work done in isothermal expansion is \( W_1 = n_1RT_1 \ln \left( \frac{3}{2} \right) \) whereas that in adiabatic expansion is

\[
W_2 = \frac{n_2RT_2 - n_1RT}{\gamma - 1}.
\]

Note that during adiabatic expansion the temperature falls to \( T' \). Also in case of adiabatic process, \( TV^{\gamma - 1} = \text{constant} \). Applying this, we get

\[
T' = T_2 \left( \frac{3}{2} \right)^{1 - \frac{1}{\gamma}}.
\]

Using the fact that \( W_1 = \frac{3}{2} W_2 \), we get the required expression.